Introducing Sliding Modes in Economics

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Abstract

We propose a novel technique for modeling Economic theories (Sliding Modes or SM**),** already widely used within control modeling in Engineering. The initial experiment seeks to explain and control the phenomenon of inflation. The two main objectives of this work are: first, contribution to mathematical Economic modeling and second, analysis of the medium-high inflation and its undesirable social consequences in Latin American countries. Here we propose SM for testing a traditional economic theory like an augmented Phillips curve in the form of a differential equations model.

The first models of variable structure with sliding mode control were developed by Emelyanov and other authors such as Utkins and Itkis in the early 1950s. Here we explain how a SM model works-its conditions and properties-in a theoretical way. Setting a monetary multiequational system, the next step is to estimate values for the parameters; for that, we have chosen the Chilean economy during the period 1985-2009. Next, simulations are held for controlling the variable inflation by means of money emission, with SM. The research ends with conclusions limited to the assumptions pertinent to the theory selected and the inherent simplification of this first simulation. Its selection is just a beginning, with the ambition of extending it to other hypothesis and theories testing.

Keywords: Mathematical Methods; Dynamic Programming; Sliding Modes; Macroeconomic Policy; Inflation Stabilization.

Paper

Introduction

The effects of high or medium rates of inflation on development and social welfare are well-known. In general economic analysis, the main concerns are: on one hand, loss of purchasing power in lower income class salaries and on the other hand, decrease of investment rate.

Inflation is a phenomenon that shows singularities depending (a) on the selected period of time and its frequency and (b) on the country under analysis. The causes of inflation on one hand and how to use monetary and economic policy for stabilizing this variable on the other is a topic widely studied in the last century but still remaining a big concern in Latin American countries, especially in Argentina and Venezuela. In this sense, several aspects of inflation have been studied (monetary, income distribution, demand-pull inflation); long and short term inflation; models with nominal rigidities and others have been proposed with respect. In relation with the role of Central Banks in the determination of price's money level and its influence in agent's expectations, Kydland and Prescott (1977) have done a fundamental research: they emphasized the credibility of Central Bank and its ability to achieve a compromise between agents influencing those policies. Cagan (1954) studied expectations for inflation of the agents, taking examples of hyperinflations. Barro and Gordon (1983) developed a model for intertemporal inconsistencies; they argued that discretionary policies of Central Bank produce an average bias over inflation.

Fisher and Seater (1993) have done a very interesting research using a bivariate ARIMA, for testing neutrality and super neutrality hypothesis in the long run. They demonstrated that the order of integration of the variables affects the restrictions of both hypotheses. They found support for super neutrality hypothesis in United States and in the hyperinflationary Germany.

Monetary Economics studies the relation between real aggregate variables and nominal variables. Nowadays modeling is basically classified in three types: models of representative agents, overlapping-generations models and a third ad-hoc category (Walsh, 2010). The *sticky prices* models in general stochastic equilibrium are the base of New Keynesian models (which could be included in the first type).

Here we will introduce a new model that is widely known in Engineering but is new for Economics, at the best of our knowledge: Sliding modes (SM), which is a type of mathematical control model.

This research is related to two main concerns of Economics science: first, to contribute to the search of an empirical support for the Economic theory in general, an objective which narrows down to finding a laboratory tool for the design for Economic policies. For this purpose, a monetary problem lends itself to experimentation. Second, to tackle inflation for the sake of countries that still suffer its consequences, objective for which it is worth revisiting Monetary Theory and looking for the mechanisms of generation of inflation and its dynamics. This paper is structured as follows: in the first part, we introduce the Sliding Modes methodology. In the second part, we specify the monetary model that we will simulate. Thirdly, the parameters of the monetary model are estimated as a SUR multiequational model. And fourthly, we run the simulation in SM and discuss conclusions.

I. What are Sliding Modes models?

SM are also called Models of Variable Structure. They have been widely applied, basically in systems subject to external perturbations, because of their resilience against issues of perturbations, modeling errors and uncertainty or ignorance of parameters (among other problems which are always potentially present).

These techniques have had an important development since the pioneer research of Utkin (1992) followed by the works of Sira-Ramirez (1989, 1996), Itkis (1976) and others. An interesting summary can be found in Liu (2012).

The advance in the use of SM in Engineering over the last years is due to the advent of technologies that allow their application not only in simulations but in real systems. Besides, what used to be a disadvantage (the presence of chattering), can now be diminished till tolerated levels; one of these solving technologies is the use of frontier layers.

Sliding Modes models are therefore, control models; basically, a control law, which changes very fast in order to lead the trajectory of the states of the system towards an arbitrary surface, specifically chosen by the researcher, keeping the trajectory on that surface at least over an interval of time. It's also an input-output model.

Setting out to explain the basics of this methodology, let's suppose that there is a differential equation like the following:

$$(1)\ a\dddot{y} + b\ddot{y} + c\dot{y} + dy = e.u$$

Where a, b, c, d and e are constants and the variables over time are $y_{(t)}$ and $u_{(t)}$. Based on (1), we define the following state variables and their derivatives:

$$x_1 = y \quad x_2 = \dot{y};\ \ x_3 = \ddot{y};\ U = u$$

Where x_1, x_2, x_3 and U are the state variables. The representation of the *states-space* (set of variables in first derivatives) will be:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -\frac{1}{a}[dx_1 + cx_2 + bx_3] + eU$$

In matrix expression, equation (1) is:

$$(2)\ \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -\frac{d}{a} & -\frac{c}{a} & -\frac{b}{a}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ e\end{bmatrix}U$$

We define a desired value, which is known in each state. In that sense, the researcher defines a value for $x_{1D}; x_{2D}; x_{3D}$; such that there will be a difference between the desired value and the value predicted by the system, that we call error:

$$(3)\ \check{x}_1 = x_{1D} - x_1$$

And similarly for x_2 and x_3. If we differentiate once each expression, we will get (2) but as a function of the error. In that way, the *vector of errors* becomes the expression of errors defined as in (3).

The *desired state* may or may not be constant, since it's a set of derivatives. Therefore, the control problem translates into making state x achieve (or follow) a desired state, which could be variant in time, even in the presence of model's imprecisions.

Existence of the sliding mode

Let's consider an autonomous system expressed as:

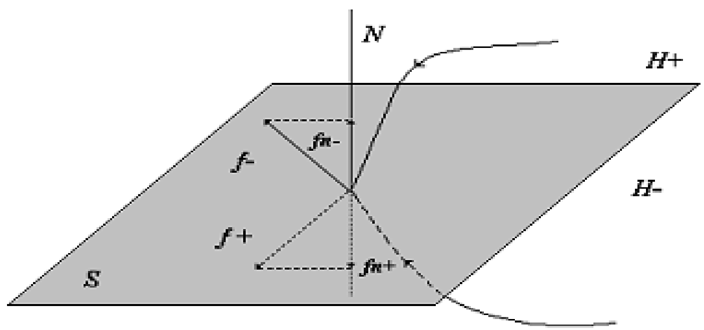
(4) $\dot{x_1} = f_i(t, x_1, x_2, \dots, x_n) \quad i = 1,2, \dots, n$

where the functions f_i are defined in the domain of the state-space, and can be considered mathematically as discontinuous functions. It can be assumed that these functions are piecewise continuous and that they present a discontinuity over the surface S defined as:

(5) $S(x_1, x_2, \dots, x_n) = 0$

If we name the state-space H, then the surface will divide it into two regions, H^+ for $S > 0$ and H^- for $S < 0$; then in a neighborhood of S, the functions f_i will be f_i^+ and f_i^-, defined in H^- and H^+ respectively, while f_N^+ and f_N^- are the respective projections on the Normal N to the surface. (Figure 1)

Figure 1: State space (3 dimensions) and sliding surface (2 dimensions). Own source.



If Filippov conditions are fulfilled, the surface will be an attractor; in that case:

$$f_N^+ < 0 \Rightarrow \dot{S} < 0$$

$$f_N^- > 0 \Rightarrow \dot{S} > 0$$

Those conditions assure that the surface will be an attractor and over it, the *sliding regime or mode* will be produced.

If the system in (1) is expressed like a controlled system *x=f(t,x,u)*, where *signal u* is discontinuous and has the form:

$$u = \begin{cases} u^+(\mathrm{t,x}) & \text{for } \mathrm{S(x)} > 0 \\ u^-(\mathrm{t,x}) & \text{for } \mathrm{S(x)} < 0 \end{cases} \qquad u^+ \neq u^-$$

Then we can extend the previous analysis for an autonomous system, to a controlled one, hence: a *sliding regime* will occur over $S(x) = 0$, if the projections of the vectors $f^+ = f(t, x, u^+)$ and $f^-(t, x, u^-)$ over the gradient of the surface S, have opposed signs and are directed towards the surface. Analytically:

$$\lim_{S \to 0+} \dot{S} < 0 \quad \text{and} \quad \lim_{S \to 0-} \dot{S} > 0$$

<u>Example 1</u>: Let's consider a system of two state variables. The perturbation is modeled as a function $F_p = a\sin(wt)$. The model in state variables is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K_1 & -K2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_3 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} F_p$$

and then a *Sliding Surface* can be defined:

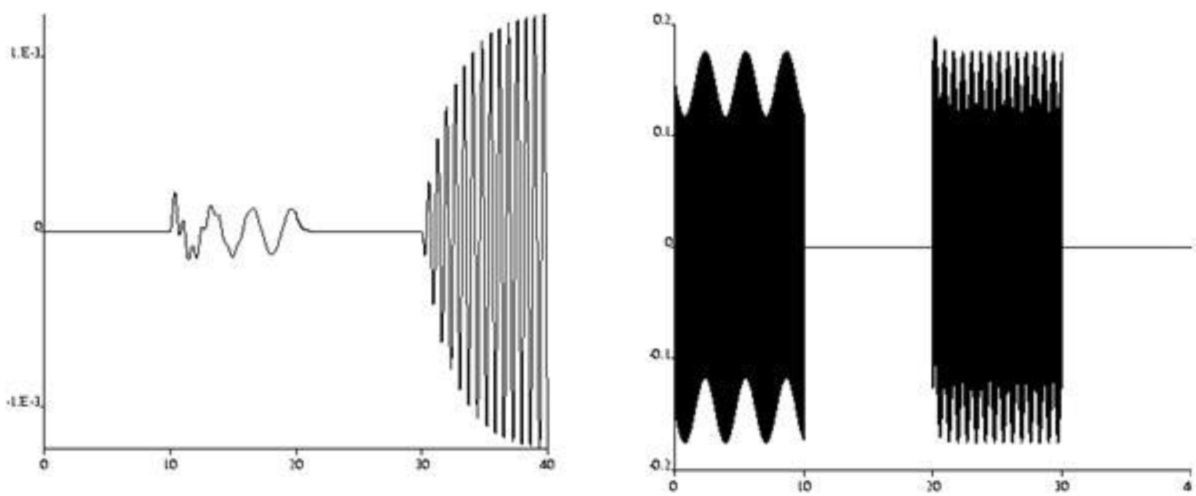
$S(t) = x_2(t) + c_1 \cdot x_1(t) = 0$ where c_1 is a design constant.

As we said before, this sliding surface must behave as an "attractor" and will be a line in the phase plane along which the states of the system will "slide" towards the origin.

The abovementioned will be fulfilled if the Liapunov function: $L = \frac{1}{2} S^2$ is defined.

The signal in which we are interested can be observed with and without control in figure 2a) and an applied control signal in 2b). When the control is applied, the output is zero, while when no control is applied, the output begins to be affected by the perturbation.

<u>Fig. 2</u>: a) Output of the system with and without control, at different frequencies b) Control signal applied.



The control in SM will be robust because the dynamic of the system is generated by the surface and because it doesn't depend on any parameter of the system. Nevertheless, for the purpose of maintaining the trajectory of the states on the sliding surface, it will be necessary to change the law of control each time that the trajectory goes through the surface. In an ideal SM, this control action will be generated with high-value infinite frequency. Because of *hysteresis*, lags and inertia of the systems encountered in real world, this frequency is finite and can excite non-modeled dynamics of the system (*chattering*). [1]

The presence of *chattering* is undesirable in the practice, because it involves very high frequency oscillations that can excite non-modeled dynamics. In that case, smoothening of the discontinuous control signal is required in order to achieve a compromise between the signal tolerated by the real physical elements and the required precision.

In a Macroeconomic model, where sample time frequency could be hourly, daily and lower (rather than of the order of magnitude of milliseconds, like in Engineering), this can be solved more easily. Chattering can be eliminated or reduced a lot, with a frontier or boundary layer.

II. The Economic Model

The main objectives of monetary policy are price stability, jobs creation and economic growth. These objectives are the subject of a wide academic debate, between supporters of neutrality and non-neutrality of money. In this work, we do not enter into the discussion about the objectives of monetary policy and the mechanisms of price-formation; instead, we take a simple hypothesis as a starting point, which could be of simple translation into a mathematical expression; with that, we will try to test the validity of that hypothesis by means of SM simulation.

Our selected model is an augmented Phillips curve, in the way of a model of 3 differential equations, which tries to explain the dynamic of the rate of inflation. This model is discussing the trade-off between inflation and unemployment, and is simple enough to be a starting point for experimentation in the use of SM models in Macroeconomics.

Friedman's hypothesis says that an inflationary trend has an extended effect, people tend to create certain expectations about inflation that they will try to incorporate into their salaries. In that way, salaries are an increasing function of inflation. Besides, Mc. Callum and Nelson (2010) pointed out the importance of re-taking the role of money emission in monetary models.

In our economy there are just only two markets: labor market and money market. [2] If the rate of inflation is a function of unemployment rate and expectations:

(6) $\pi_{(t)} = \alpha - \beta U_{(t)} + hE_{(t-1)}\pi_{(t)}$; with $\alpha,\ \beta > 0$

Where: π: rate of inflation, calculated on level of prices. U: unemployment rate (net of productivity effect); $E_{t-1}\pi$: expected level of inflation (Friedman, 1968). Then (6) is a Phillips' curve.

If we think about inflation's expectations as adaptive (Cagan, 1954), we can write:

(7) $\frac{dE_{(t-1)}\pi_{(t)}}{dt} = j(\pi_{(t)} - E_{(t-1)}\pi_{(t)})$; with $0 < j < 1$

For the unemployment dynamic behavior:

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[1] For more advanced texts in SM methodology, consult Sira Ramirez & Llanes Santiago (1994); Slotine and Weipig (1991) and Utkin (1992).

[2] The model was taken from Alpha Chiang (2005), slightly modified and solved by the authors of this paper. Also, its discrete form had to be translated into continuous time for using SM.

(8) $\frac{dU}{dt} = -k(M - \pi_{(t)})$;   with $k > 0$

Where: M: rate of growth of nominal money. Then, the right side of (8) is real money growth, multiplied by a constant. This equation specifies a negative relationship between unemployment variation and real money growth. At the same time, the system shows the feedback between inflation and unemployment.

Then, our model of three endogenous variables $\pi, E_{t-1}\pi, U,$ is defined by (6), (7) and (8). We solve the system for π, keeping M as the control variable, obtaining the following ordinary differential equation:

$$(9)\ \frac{d2\pi}{dt2} = [\beta ka + hj - \beta k - j]\frac{d\pi}{dt} - (j\beta k)\pi + j\beta kM$$

Equivalently:

$$(9`)\ \ddot{\pi} = k_2\dot{\pi} + k_1\pi + k_3M$$

Where the stability of the system needs k_2 and $k_1$negative. From the analysis of the roots of the characteristic equation, we conclude that the system is stable. In (9) and (9’), the variable to be controlled is the rate of inflation and the manipulator variable is nominal money (like in quantitative theory).

Now that we have our economic model, let’s work in the input-output or control model. We transform the previous system into a state-variables one, defining:

$$(10)\qquad x_{1(t)} = \pi_{(t)};\ x_{2(t)} = \dot{\pi}_t;\ u_{(t)} = M_{(t)}$$

We are interested in controlling $\pi_{(t)}$ by means of the manipulation of $M_{(t)}$; or $x_{1(t)}$ and $u_{(t)}$, respectively. Equation (1) can be written as:

$$(11)\qquad \dot{x}_{2(t)} + a_1x_{2(t)} + a_2x_{1(t)} = b.u_t$$

where u is the control signal. If we solve for $\dot{x}_{2(t)}$ in matrix expression:

$$\left[\dot{x}_{2(t)} = -a_1.x_{2(t)} - a_2.x_{1(t)} + b.u_{(t)}\right]$$

Finally, the model in state-space results:

$$\begin{pmatrix}\dot{x}_{1(t)}\\ \dot{x}_{2(t)}\end{pmatrix} = \begin{pmatrix}0 & 1\\ -a_2 & -a_1\end{pmatrix} * \begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ b\end{pmatrix} * u_t$$

The expression for the model in state-space plus the output of the system is:

$$\left[\dot{x}\right] = [A][x] + [B]u$$
$$[y] = [C][x]$$

Where A is a matrix while B and C will be vectors. The first equation is the *state-equation* and contains the dynamic of the system, while the second one is the *output equation.* This model has still no perturbation, which will be included in the simulation step, and could be added to the state-variables or to the output variable. In our simulation, we included one perturbation into the output variable. $\dot{x}$ is equals to the sum of $A.x + B.u$; by integrating that, x is obtained, and by multiplying it by C, the output y is obtained.

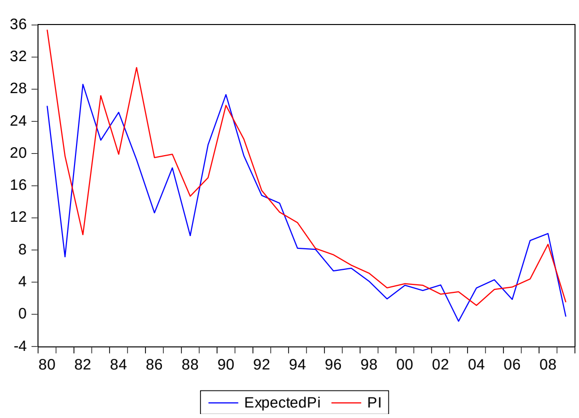
Again, nominal money is the input of the system and rate of inflation is the output.

III. Econometric estimation of parameters and simulation in SM

Next step is to give values to the parameters or constants of our Economic model. For that, we will run an econometric estimation, based on data of a Latin American country. But for a first experiment, we need data from a relative stable period and economy. Because of their macroeconomic performance in the last decade, possible candidates are Brazil, Uruguay and Chile. Finally, Chile is selected. The estimation will have annual frequency, that is, the estimated values will describe a long-run relationship.

Since there is no available measurement for the expected rate of inflation a proxy variable is needed for the calculation of which there are different alternatives. The first alternative is a survey of the Central Bank of Chile, which however takes into account the expectations of merely businessmen and academics who represent only a small part of the population. For similar reasons, the inflation compensation from financial market was equally discarded. Finally, we calculated a proxy, based on a moving average of eight monthly lags, taking in account seasonality. The proxy is stationary and explains 67% of the actual inflation. The first derivative of this proxy follows Cagan's adaptive expectations described by equation (7)[1].

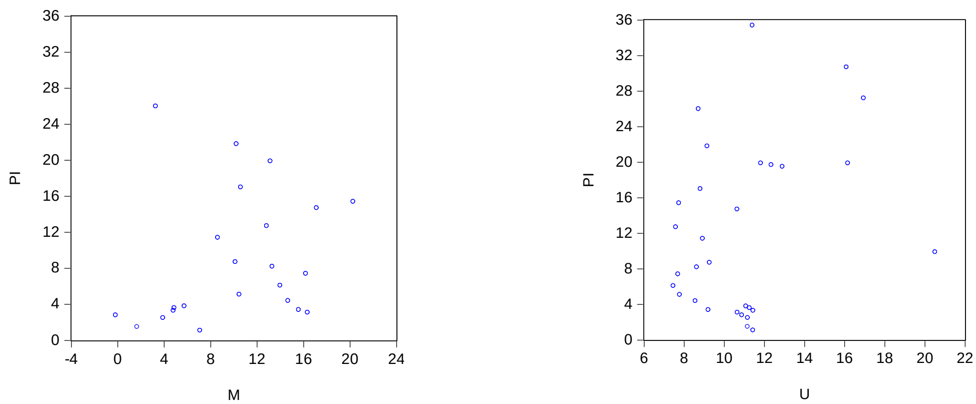
Figure 3: Inflation and expected inflation. Chile, annual, 1985-2009. Based on CEPAL



[1] The first derivative is taken to be roughly equal to its discrete version by taking first differences of the variable.

A first simple graphical analysis of correlation between π and M suggests a non-linear correlation between both (figure 4 a). Figure 4 b) explores Phillips' curve shape. Following Friedman, in the short-run, its slope is negative, while in the long-run, a vertical curve could appear and even a positive slope. In figure 4 b), there is not a clear relationship, if any at all.

<u>Figure 4: Chile, annual, 1985-2009, based on CEPAL. a) Relationship between π and M. b) Phillip's Curve.</u>



Now we estimate the parameters for the model presented in (6), (7) and (8). As vector autoregression models assume all variables and certain amount of lags as endogenous, we use Seemingly Unrelated Regression (SUR), which accounts for autocorrelation in the disturbances. Another reason of choosing this technique is the introduction of the less possible assumptions in the Econometric stage, so that the simulation could be kept free of specification errors as much as possible. The estimation surpassed tests of autocorrelation and heteroscedasticity. Instrumental variables in two stage least squares were also tried, which did not show so much difference in the estimated values. What is more important, the signs and range of the variables behaved as expected.

In the next table, estimated parameters for the long-run are shown, as much as their significance levels.

<u>Table 1. Estimation of parameters by SUR</u>

Number of equation	*Parameter*	*Estimated value*
(6)	α	14.45
	β	0.35
	h	0.69
(7)	j	0.06
(8)	k	0.13

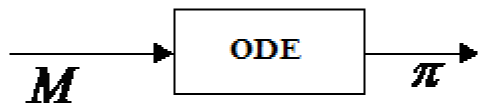
(a) All the estimated coefficients are significant at 5 and 10%, except for j. (b) Some variables needed to be stationarized. (c) The residuals of the system have no autocorrelation according to Portmanteau test.

Simulation in SM

As next step in determining the ordinary differential equation that summarizes the dynamic of the whole system, we assumed that $\dot{M} = a.\dot{\pi}$, where a is a constant of proportionality positive and less than 1. [1]

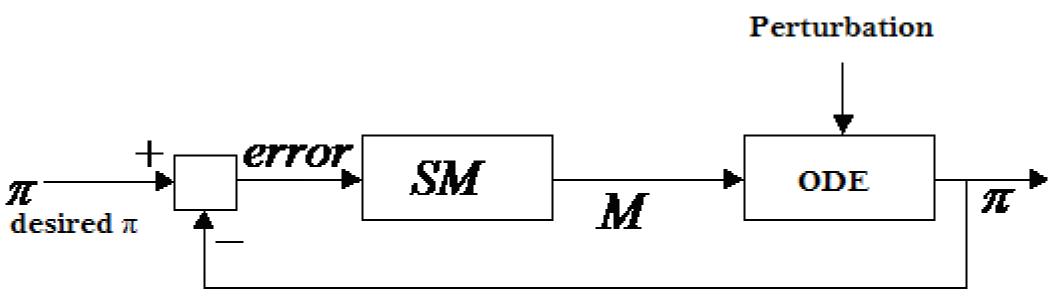
a) First step and trial: we assign the value 1 to M, and we watch what happens with the output without using SM. This step serves only for verifying the stability of the system. (Figure 5)

Figure 5. Block diagram for step 1. Own source.



b) Second step: given that in step 1 we got a good behavior, we operate with SM. Doing so, the system was perturbed with a white noise signal, which at the same time allows testing the robustness of the mathematical model. Really, what we want is to make zero the error between the desired value of pi, and the real value. We chose a desired value to pi (2%.) We assign a value 1 to the input *M*, for watching its impact on output π. Sliding modes and the control variable *M,* conduct the path of pi to make the error zero.
   The white noise disturbance and *M* enter into the ordinary differential equation at the same time (figure 6).

Figure 6. Block diagram for the 2nd step. Own source

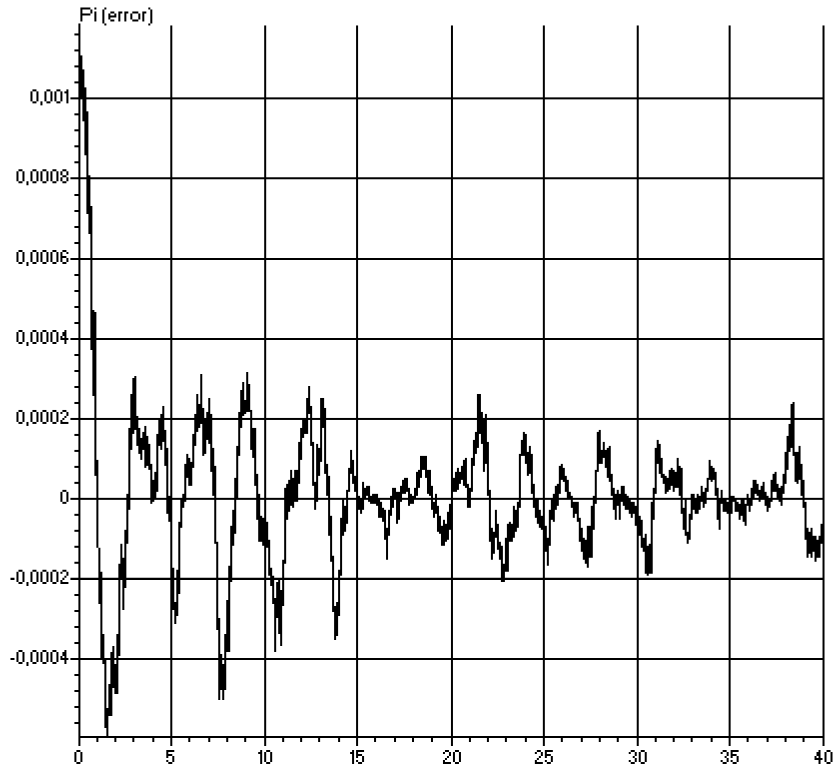


The output of the system π can be controlled. In figure 6, we show how the error (the difference between the desired and the real π) tends to zero before the time unit 20. In each period, this difference is corrected by SM, by means of modifying *M,* in order to obtain an error equal to zero. And that's the reason of the name of the model: the system "slides" on the surface till the

[1] The estimated value for *a* with the mentioned data, is 0.92.

origin (zero). So whatever the value we assign to desired π, SM will try to make the error equal to zero. [1]

Figure 7: Error on variable π (Pi desired minus Real Pi)



The oscillations occur because the characteristic equation of (9) has complex conjugated roots. But they tend to zero over time.

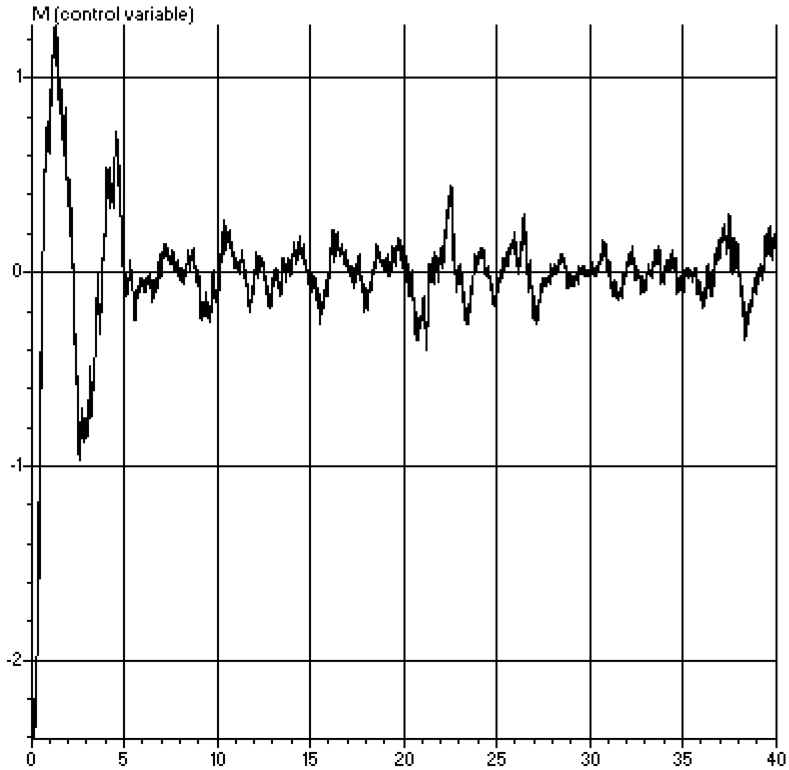
In the error graph, what will be important to us are the maximum and minimum values taken by the variable. They must tend to zero; in that way, the input is efficient for getting a desired value of the output. In the first period, of course, the error will not be zero.

Let's now watch the behavior of *M* in Figure 8. We can see that the manipulation of *M* that we have operated is not extreme, and like π oscillates around zero. The economic interpretation of the latter is the following: the system departs from repose; SM injects movement, that is, changes the rate of growth of money, and taking account of all the other variables and relationships that we have designed, is able to conduct π to the desired value, touching and crossing surface *S*, in another word, *sliding* over it.

Graph in Annex shows in a comparative way the trajectories of the error in π, *M*, the trajectory on the surface *S*, and the error for $\dot{\pi}$.

[1] The simulation added a technique called "frontier layer". The explanation of this methodology exceeds the objective of the present paper. But the reader can consult the more advanced texts on SM cited in Part I. The frontier layer is used for making the impact of *M* smoother.

Figure 8: Manipulation of *M*.



IV. Discussion of results and Conclusions

The monetary model of 3 equations could be represented as a control SM model with frontier layers. With π as the control variable and M as the control signal, the model is stable. In any system stable in all its versions, modeled by SM, the version that should be selected is the one that is applicable from the point of view of the control signal; in our case, M must be applicable. In other words, and according to its scale, M should be able to be introduced without abrupt oscillations. The output of the system turns out to be a controlled one. U and $E_{t-1}\pi$ could not be controllers from the point of view of Economic theory: they are not controllers or manipulators in the real world.

We can use the output that we obtained as an empirical laboratory for testing theoretical hypothesis, like Friedman's corollary: if an inflationary trend has an extended effect, workers will tend to create expectations about the rate of inflation that they will later try to incorporate into their salaries. Thus, salaries are an increasing function of inflation. The sign of the relationship between inflation and employment depends on the frequency of the observations (short or long-run) and on the variables involved in an augmented Phillips' curve. The negative relationship is likely to arise in the short run, while in the long-run, a vertical slope curve or even a positive slope, could be expected. In the long run, workers and employers will take into account the rate of inflation in the moment of negotiating the value of salaries and this can push both variables in the same direction.

The previous observation will also depend on the magnitude of the rate of inflation (including also expected inflation) and the unemployment rate indicated by the market. With high rates of unemployment, this variable will be more inelastic to changes in salaries. In the case of our sample of thirty annual observations, the first 10 years show unemployment rates above 9% and inflation rates above 15% (values a bit higher than expected for a healthy stable economy, but still valid for our experiment).

The present research accounts for long-term data. The graph shows no clear correlation between inflation and unemployment; when we define a system in which we add more variables from both labor and money markets the Econometrics tell us that the slope of augmented Phillips' curve is negative. And if we use this information to model the system as a SM, then SM confirms the Econometrics, not only in relation to signs, but also in relation to the functions that interlace the variables. Why? Because if we can get a stable output for a dynamic system, it means that our model represents the multiple local equilibriums that conform the path of rate of inflation, in its dynamic. Real life shows us a path for the variable of interest (rate of inflation) with multiple equilibriums along time. We introduce this design in SM and SM gives a controlled output, using money as a manipulator. If the output variable can be controlled, that is because the model represents in an acceptable way the dynamic of the rate of inflation, given the variables that were taken. According to Nelson and Mc Callum, money matters: this hypothesis is also confirmed by SM.

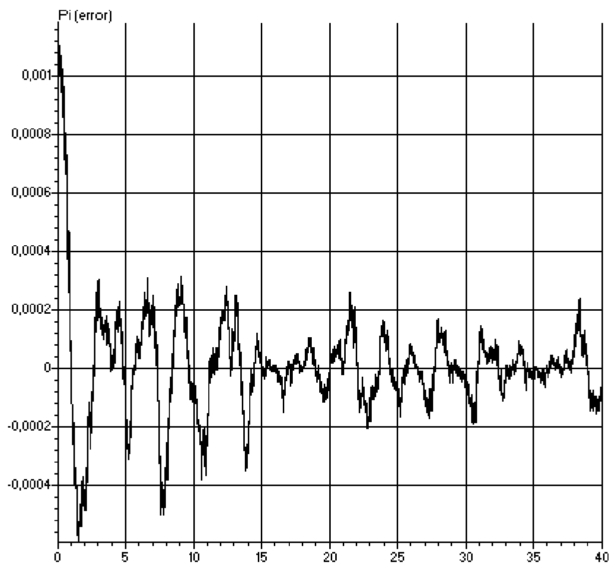
Of course, a variable like interest rate should be considered. The effect of variables that has not been considered in this simple model, is accounted for by the other variables that has been included. This happens in every econometric model. Again, for making a first trial of SM, a very simple model has to be taken.

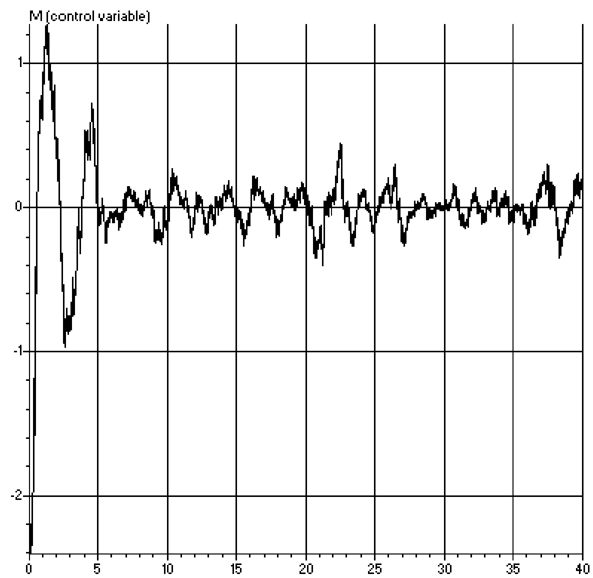
The SM simulation, with values as the ones estimated for Chile, confirms:

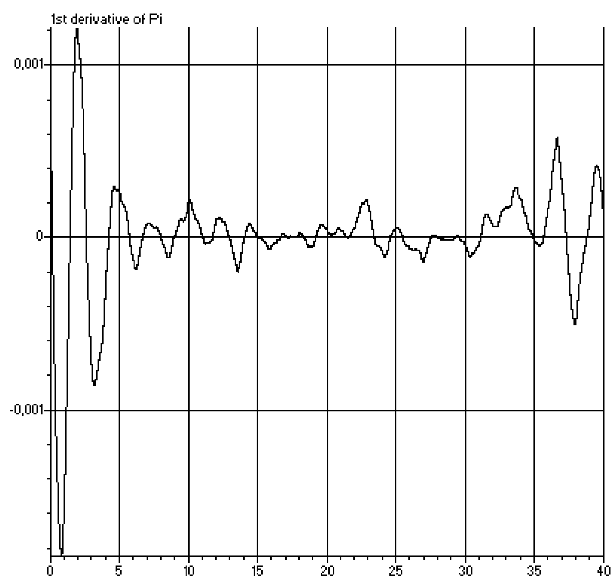
a) a negative slope in the long run for the augmented Phillips' curve;
b) a positive effect of injection of money on rate of inflation
c) salaries are an increasing function of inflation.

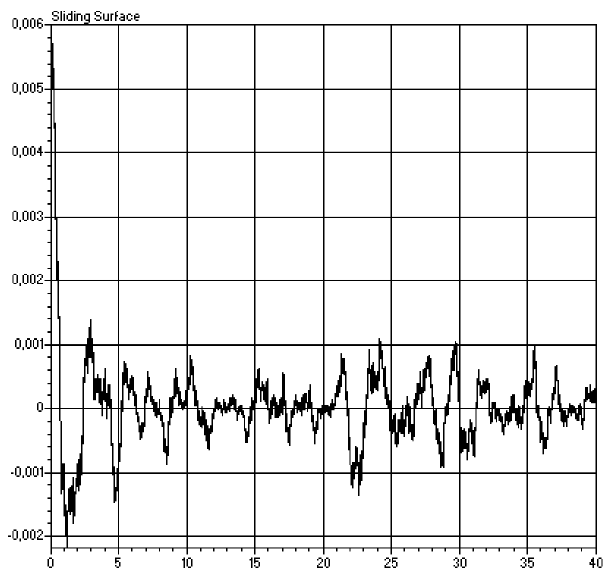
Overall, SM proves to be as a valid empirical alternative for testing macroeconomic hypothesis and theories. A future step would be to try this model as a laboratory tool for calibrating economic policies. This preliminary research that we have presented here aimed to be a first step for the future use of this model in our discipline.

Annex: Comparative graphs of the simulation: error in pi, M, 1st derivative of pi and sliding surface









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